

# Enhanced gravitational scattering from large extra dimensions

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## Abstract

We investigate whether enhanced gravitational scattering on small scales ( $< 0.1\text{mm}$ ), which becomes possible in models with large extra dimensions, can establish statistical equilibrium between different particle species in the early Universe. We calculate the classical relativistic energy transfer rate for two species with a large ratio between their masses for a general elastic scattering cross section. Although the classical calculation suggests that ultra-light WIMPs (e.g., axions) can be thermalized by gravitational scattering, such interactions are considerably less efficient once quantum effects are taken into account on scales below the Compton wavelength. However the energy transfer rate in models with several extra dimensions may still be sensitive to trans-Planckian physics.

# 1 Introduction

Higher dimensional models suggest a phenomenologically interesting solution to the hierarchy problem. A higher dimensional Planck mass  $M_D$  just above the electroweak scale is in fact compatible with the observed weakness of four-dimensional gravity ( $M_P \sim 10^{19} \text{ GeV}$ ) if we consider relatively large compactification radii. In braneworld models where gravity freely propagates in  $4 + d$  dimensions [1] and the  $d$  internal dimensions share the same compactification radius  $R$ , the latter is related to  $M_D$  as

$$R \simeq \left( \frac{M_P}{M_D} \right)^{2/d} M_D^{-1} \simeq 10^{16(2/d-1)} \left( \frac{\text{TeV}}{M_D} \right)^{1+2/d} \text{mm}. \quad (1)$$

Within length scales of order  $R$  gravity is genuinely  $4 + d$  dimensional and the gravitational potential starts growing as  $\sim 1/r^{1+d}$ . This rules out  $M_D \sim 1 \text{ TeV}$  for  $d = 1$  models, while leaving a small window for  $d = 2$  [2]. A fundamental Planck mass of  $\sim \text{TeV}$  opens the intriguing possibility of recording quantum gravity signals at the next generation of accelerators, where those energy scales will be actually probed [3].

In this note we investigate whether stronger gravity on sub-mm scales could affect the thermal history of the early Universe due to the enhanced gravitational scattering cross-section predicted by these models, even at energies and temperatures much less than the electroweak scale. Below the quantum gravity regime, we consider relativistic scattering processes between species of very different masses  $m \ll M$  in the limit of small scattering angle/low momentum transfer and estimate the efficiency of such processes for establishing statistical equilibrium between species. More specifically, we will consider the heavy species as relativistic and in thermodynamical equilibrium with an average energy per particle  $\sim T$  and the light species initially decoupled and “cold” i.e. with an energy per particle  $\ll T$ . This is the case of the QCD axions, which acquire an effective mass  $m \simeq 10^{-5} \text{ eV}$  at temperatures as high as a GeV.

Ultra-light weakly interacting massive particles (WIMPs) such as the axion are candidates for the cold dark matter (CDM) that is responsible for around 25% of the energy density of the Universe today and play a central role in cosmological structure formation. A crucial requirement for these ultra-light WIMPs is that they are non-thermal. In particular the QCD axions [4] arise from coherent oscillations about the minimum of its potential of the spatially homogeneous axion field that begin when the temperature drops below about 1 GeV at the QCD phase transition. They are supposed to be effectively non-interacting at this temperature and thus remain at rest until local overdensities undergo gravitational collapse, beginning the process of structure formation, from the bottom up. Today axions are supposed to be present in the virialised dark halo surrounding visible stars in galaxies, with velocity dispersion of order  $100 \text{ km s}^{-1}$ .

The gravitational cross-section of ultra-light WIMPs, enhanced by the presence of large extra dimensions, has recently been proposed [5] as a natural mechanism for the CDM “self-interaction”, advocated in [6] to resolve the “cusp problem” [7] in the centre of galaxies (other proposed solutions are found e.g. in [8]). In this paper we consider another effect of the enhanced gravitational scattering. We consider whether gravitational scattering between ultra-light WIMPs and much heavier but relativistic species in the early universe (such as electrons or neutrinos) could establish a statistical equilibrium between different species at energies well below the fundamental Planck scale. If the energy transfer is sufficient to make the axions highly relativistic, then they would no longer be viable candidates for the CDM in models with large extra dimensions.

## 2 Classical gravitational scattering in higher dimensions on a brane

In the rest frame of the heavy particle, the light particle feels a central attractive gravitational force

$$|\mathbf{F}(r)| = \frac{Mm\bar{\gamma}}{M_D^{d+2}r^{d+2}}, \quad M_D^{-1} < r < R, \quad (2)$$

where  $\bar{v}$  is the speed of the light particle in this frame, and  $\bar{\gamma} \equiv (1 - \bar{v}^2)^{-1/2}$  is the usual relativistic Lorentz factor. Note that we use this equation as our definition of  $M_D$ . Equation (2) is derived from the  $4 + d$  dimensional propagator and the lower bound  $r > M_D^{-1}$  is the limit of validity of such a tree-level calculation. Above the compactification radius,  $r > R$ , on the other hand, the usual Newtonian inverse-square law is restored (or  $r > \ell_{AdS}$  in the Randall-Sundrum model [9] with an Anti-de Sitter bulk if  $R > \ell_{AdS}$ ).

In the small scattering angle limit the transverse momentum  $p_T$  acquired by the light particle can be estimated as

$$p_T = \frac{2b}{\bar{v}} \int_0^\infty dx \frac{|\mathbf{F}|}{\sqrt{b^2 + x^2}} \simeq \frac{\bar{\gamma}}{\bar{v}} \frac{Mm}{M_D^{d+2}b^{d+1}}, \quad (3)$$

where  $b$  is the impact parameter. It will be useful to define the velocity-independent dimensionless parameter

$$\epsilon = \frac{\bar{v} p_T}{\bar{\gamma} m} = \frac{M}{M_D^{d+2}b^{d+1}} \quad (4)$$

that, in the relativistic limit  $\bar{v} \simeq 1$ , is just the transverse momentum in units of the energy of the particle. As long as we consider impact parameters bigger than the Planck scale cut-off  $M_D^{-1}$ , we have  $\epsilon < M/M_D \ll 1$ .

In Appendix A we calculate the energy transferred to ultra-light WIMPs due to gravitational scattering with much heavier, but relativistic particles such as electrons with Lorentz factors  $\gamma \gg 1$  in the cosmological reference frame. We find, from Eq. (22), an energy transfer rate

$$\frac{dE}{dt} \simeq \pi n_* E [\gamma^2 - 1] \int_{b_{UV}}^{b_{IR}} db b \epsilon^2, \quad (5)$$

where  $n_*$  is the number density of the heavy particles and  $\epsilon(b)$  is given by Eq. (4). The energy transfer rate in higher-dimensional gravity [ $d > 0$  in Eq. (4)] is dominated by scattering events with small impact parameters due to the steep rise in the gravitational force on small scales  $r \ll R$ . Thus the energy transfer is not sensitive to the IR cut-off,  $b_{IR}$  (here given by the compactification scale  $\sim R$ ), but is sensitive to the UV cut-off,  $b_{UV}$ . Note that in 4 dimensional theory ( $d = 0$ ) the energy transfer depends on the cut-off scales only logarithmically  $\log(b_{IR}/b_{UV})$ .

We must cut-off our perturbative calculation at least at the fundamental Planck scale,  $b_{UV} \sim M_D^{-1}$ , which is the limit down to which we can trust Eq.(2). Note that for elementary particles with  $M < M_D$ , the Schwarzschild radius  $r_S \sim M_D^{-1}(M/M_D)^{1/(d+1)}$  of the heavy particle is much smaller than  $M_D^{-1}$ . A more serious problem is that our result is sensitive to trans-Planckian scattering and a reliable calculation requires knowledge of non-perturbative quantum gravity on smaller scales! *The classical energy transfer rate due to gravitational scattering in brane-world models is sensitive to trans-Planckian physics.*

On the other hand the Compton wavelength of our particles are much larger than  $M_D^{-1}$  so a full calculation requires a quantum mechanical treatment. We estimate the effect of quantum corrections in the following section, but in the rest of this section consider the effect of classical scattering.

We can estimate the classical rate of energy transfer to the ultra-light particles as

$$\frac{dE}{dt} \simeq \pi n_* E [\gamma^2 - 1] \int_{M_D^{-1}}^R b \epsilon^2 db \simeq \frac{\pi}{2d} \frac{ET^2 n_*}{M_D^4}. \quad (6)$$

where we take the heavy species to be relativistic ( $v \simeq 1$ ) and in thermodynamical equilibrium ( $\gamma M \simeq T$ ). Note that this result is independent of the rest mass of the heavy particles and depends only upon the number density of relativistic particles.

Thus we would expect a rapid rise in the energy of ultra-light WIMPS due to classical gravitational scattering at sufficiently early times when  $\dot{E}/E \gg H$ . Note that Eq. (6) has been derived in the limit  $\bar{\gamma}m \ll M$ , which was used in Appendix A to assume negligible recoil of the heavy particles during each collision. In practice the heavy particles are not infinitely heavy, and hence are not an infinite energy source, and the energy transfer rate must decrease as statistical equilibrium is approached.

From (6) the relaxation rate of our light species can be expressed as a function of the temperature by taking the number density  $n_* = 1.8\mathcal{N}_f T^3/\pi^2$  of fermionic relativistic species,

$$\frac{\dot{E}}{E} = 10^{-12} \frac{\mathcal{N}_f}{\pi d} \left( \frac{\text{TeV}}{M_D} \right)^4 \left( \frac{T}{\text{GeV}} \right)^5 \text{GeV}, \quad (7)$$

$\mathcal{N}_f$  being the effective number of relativistic fermionic species. Gravitational interactions can efficiently establish equilibrium if, at some epoch in the early Universe,  $\dot{E}/E \gg H$ , where the Hubble rate in the radiation dominated era is given by

$$H \simeq 10^{-18} \left( \frac{T}{\text{GeV}} \right)^2 \text{GeV}. \quad (8)$$

A more detailed calculation (see Appendix B) shows that the light particles acquire enough energy so that they are still relativistic at the beginning of matter domination  $T_{\text{eq}} \sim 10 \text{ eV}$ , if  $\dot{E}/E > \alpha H$ , where  $\alpha \simeq 30$ . In this case even if the light particles are initially “cold” when produced at  $T_A$ , they soon become relativistic due to the gravitational scattering by heavy relativistic particles in thermal bath. They remain relativistic until matter-radiation equality if

$$\frac{\mathcal{N}_f}{\pi d} \left( \frac{T_A}{\text{GeV}} \right)^3 > 10^{-2} \alpha \left( \frac{M_D}{10 \text{ TeV}} \right)^4. \quad (9)$$

Thus, for  $M_D \lesssim 10 \text{ TeV}$ , axions produced at  $T_A \sim \text{GeV}$  would acquire sufficient energy that, even by the time of matter-radiation equality, they are still relativistic and would therefore be ruled out as dark matter candidates.<sup>1</sup>

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<sup>1</sup>Note that our assumption of elastic scattering of the ultra-light particles in the rest frame of the heavier particles required  $\bar{\gamma}m < M$  which corresponds to  $E < M^2/T$ . For instance considering gravitational scattering with electrons at  $T \sim 1 \text{ GeV}$  our calculation requires  $E/T < 10^{-6}$ . This is just sufficient to show that axion with mass  $m \sim 10^{-5} \text{ eV}$  remain relativistic at  $T_{\text{eq}} \sim 10 \text{ eV}$ . In practice we don’t expect energy transfer to shut-off completely for  $M^2/T < E < T$ , but it may become less efficient.

The reason why gravitational scattering remains efficient far below the fundamental Planck scale is the hierarchy between the fundamental Planck scale and the effective four-dimensional Planck scale,  $M_P \sim 10^{19}$  GeV, which determines (in practice, suppresses) the Hubble expansion rate at these energies. For models where the fundamental Planck scale is as low as a TeV, gravitational scattering of ultra-light WIMPS remains efficient down to temperatures as low as 10 MeV. This is analogous to the neutrinos remain coupled to baryons down to energies of order 1 MeV even though the electroweak scale is of order 100 GeV.

### 3 Quantum effects

In calculating the energy transfer in the previous section we considered the particles as point-like and the scattering as a completely classical processes. By taking  $b_{UV}$  in (5) as  $M_D^{-1}$  we have implicitly assumed that the two particles can come arbitrarily close to each other, neglecting their finite Compton wavelengths. In the mass frame of the heavy particle the Compton wavelength of the light particle is  $(\bar{\gamma}m)^{-1}$ . By taking  $\bar{\gamma}$  as given by (14) and by averaging over the angle  $\theta$  we can estimate the Compton wavelength of the light particle as  $\sim M/(ET)$ . By taking this rather than the Planck scale as the small scale cut-off  $b_{UV}$  in (5) we obtain the energy transfer

$$\frac{dE}{dt} \simeq \frac{E^{2d+1}T^{2d+5}}{M^{2d}M_D^{2d+4}}, \quad (10)$$

here we assume  $R > (\bar{\gamma}m)^{-1}$  so that we still probe the higher-dimensional gravity. The relaxation rate  $\dot{E}/E$  for axions is now much smaller than the Hubble rate  $H$  for  $M_D \sim$  TeV:

$$\frac{\dot{E}/E}{H} \simeq \frac{M_P E^{2d} T^{2d+3}}{M^{2d} M_D^{2d+4}} \simeq 10^{7-6d} \left( \frac{T_A}{GeV} \right)^{2d+3} \left( \frac{TeV}{M_D} \right)^{2d+3} \left( \frac{m}{M} \right)^{2d}. \quad (11)$$

The last equality holds at  $T = T_A$  when the energy of the axions is  $E \simeq m$ .

Rather than introducing an abrupt cut-off at the Compton wavelength, we can try to estimate the suppression due to the interference of the wavefunction for impact parameters smaller than the Compton wavelength,  $l_C$ , of the particle. In the case of a uniform potential well of radius  $a$  the quantum mechanical cross section is suppressed, with respect to the classical one, by a factor  $(a/l_C)^6$  [10], i.e.  $\sigma_{\text{quantum}} = \sigma_{\text{classical}}(a/l_C)^6$ . The cross section has a direct interpretation in terms of “number of particles scattered”. Therefore, in the integral in (6), when considering impact parameters smaller than the Compton wavelength,  $b < l_C$ , we should include the corresponding suppression factor:

$$\frac{dE}{dt} \simeq \pi n_* E [\gamma^2 - 1] \int_{M_D^{-1}} \left( \frac{bTE}{M} \right)^6 b \epsilon^2 db \simeq \frac{E^7 T^{11}}{M^6 M_D^{2d+4}} \int_{M_D^{-1}} b^{-2d+5} db. \quad (12)$$

Note that, as recently emphasized also in [11], for  $d \geq 3$  the steepness of the gravitational potential wins over the quantum suppression effect, and the process is still sensitive to trans-Planckian physics. For  $d < 3$ , on the other hand, most of the contribution comes from impact parameters larger than the Compton wavelength. Note also that for  $d \geq 3$  the energy rate is given by (10) with  $d = 3$ .

## 4 Conclusion

In this letter, we studied the impact of an enhanced gravitational scattering cross-section in the early universe in the context of the brane-world models with a low fundamental Planck scale. In the presence of a general interaction of some given elastic scattering differential cross section, we calculated the relativistic energy transfer rate for two species with a large ratio between their masses. Due to the enhanced gravitational scattering cross section, ultra-light WIMPs could reach a statistical equilibrium with heavier relativistic particles in the thermal bath even well below the fundamental Planck scale. For example, the QCD axions produced at  $\sim 1$  GeV could soon become relativistic and they remain relativistic until matter-radiation equality due to classical gravitational scattering. Axions are usually supposed to be non-interacting because their interactions are suppressed due to the existence of a much larger mass scale  $f_a \sim 10^{12}$  GeV. But in models with large extra dimensions their gravitational interactions can only be suppressed by the fundamental Planck scale,  $M_D \ll f_a$ .

However the classical energy transfer is dominated by scattering events with impact parameters much less than the Compton wavelength of the light particles. If we cut-off our classical calculation at the Compton wavelength the effect disappears completely. If instead we attempt to model the quantum suppression of the calculation below the Compton wavelength we find that for  $d > 3$  the result remains sensitive to the ultra-violet cut-off of the theory at the fundamental Planck scale [11]. This is due to the steep rise in the gravitational force on small scales in higher dimensional spacetime. Thus, in a higher dimensional theory, the energy transfer due to gravitational scattering is sensitive to trans-Planckian physics.

It is interesting to consider whether other phenomena could be sensitive to enhanced gravitational scattering. We commonly neglect the effect of gravitational scattering as we expect it to be weak compared to all other interactions. But in models with large extra dimensions gravity can be much stronger on small scales than we naively imagine. And in contrast to four-dimensional gravity, the scattering is sensitive to trans-Planckian physics. Even the absence of detectable gravitational interactions could place constraints upon gravitational scattering on sub-Planck scales.

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## A Energy transfer rate by classical relativistic scattering

We want to study the energy transfer between two particle species, with masses  $m_1 \gg m_2$ , in terms of classical relativistic scattering. We assume that in the cosmological reference frame (e.g. the rest frame picked out by the cosmic microwave background) these species have speeds  $v_1$  and  $v_2$ , and that in this cosmic rest frame the velocity distribution of both species are isotropic.

A given scattering event is characterized by the angle  $\theta$  between the initial 3-velocities  $\mathbf{v}_1$  and  $\mathbf{v}_2$  of the two particles considered. Without loss of generality we can write the initial four-momenta in the cosmic rest frame as  $m_1\gamma_1(1, v_1, 0, 0)$  and  $m_2\gamma_2(1, v_2 \cos \theta, v_2 \sin \theta, 0)$  respectively, where  $\gamma_1$  and  $\gamma_2$  are the usual relativistic Lorentz factors.

In the particle-1 rest frame (RF1), the four-momentum of particle 2 is

$$\bar{p}_{\text{ini}} \equiv (\bar{E}, \bar{p}_x, \bar{p}_y, \bar{p}_z) \equiv m_2 \gamma_2 (\gamma_1 [1 - v_1 v_2 \cos \theta], \gamma_1 [-v_1 + v_2 \cos \theta], v_2 \sin \theta, 0), \quad (13)$$

from which the Lorentz factor  $\bar{\gamma}$  and relative velocity  $\bar{v}$  in the RF1 frame are derived as

$$\bar{\gamma}(\theta) \equiv \frac{1}{\sqrt{1 - \bar{v}(\theta)^2}} = \gamma_1 \gamma_2 (1 - v_1 v_2 \cos \theta). \quad (14)$$

In RF1 we can characterise the geometry of each scattering event by the two parameters  $b$  and  $\Phi$ . We assume that we know the scattering deflection angle  $\Theta(b, \bar{v})$  in RF1 as a function of the impact parameter  $b$  and velocity  $\bar{v}$ . (In the case of interest in this paper  $\Theta \simeq \epsilon/\bar{v}^2 \ll 1$  and  $\epsilon$  is given in equation (4) as a function of  $b$ ). After scattering, the light particle can acquire an orthogonal velocity component. We define an angle  $\Phi$  as the angle between the plane  $x - y$  and the plane containing 3-velocity of the light particle after scattering as well as particle 1. Only for  $\Phi = 0$  or  $\Phi = \pi$ , the particle is scattered on the  $x - y$  plane and in general its velocity acquires an orthogonal component proportional to  $\sin \Phi$ . The final four-momentum after an elastic collision in RF1 is thus given by

$$\bar{p}_{\text{fin}} = \left( \bar{E}, \bar{p}_x \cos \Theta + \bar{p}_y \sin \Theta \cos \Phi, \bar{p}_y \cos \Theta - \bar{p}_x \sin \Theta \cos \Phi, \sqrt{\bar{p}_x^2 + \bar{p}_y^2} \sin \Theta \sin \Phi \right) \quad (15)$$

Because we are working in the limit  $m_2/m_1 \rightarrow 0$  we require that the final speed of particle 2 is the same as its initial speed in RF1 and only its direction is changed. Total momentum is conserved due to the recoil of particle 1, but the kinetic energy transferred in RF1 is negligible as  $m_2/m_1 \rightarrow 0$ .

For each particle of type 1 the differential rate of such an event is

$$d\bar{\Gamma}_1(b, \theta, \Phi) = b \bar{v}(\theta) d\bar{n}_2(\theta) db d\Phi, \quad (16)$$

where  $d\bar{n}_2$  is the RF1-number density of those type 2 particles whose direction in the CMB reference frame is comprised between  $\theta$  and  $\theta + d\theta$ .

By Lorentz-boosting back to the cosmic rest frame we obtain the total energy of the light particle after such an event:

$$E(b, \theta, \Phi) = m_2 \gamma_1 \left[ \bar{\gamma} + \left( \frac{\gamma_2}{\gamma_1} - \bar{\gamma} \right) \cos \Theta + v_1 v_2 \gamma_2 \sin \Theta \cos \Phi \sin \theta \right]. \quad (17)$$

Thus the energy acquired after the scattering,  $\Delta E \equiv E - m\gamma_2$ , in the limit of small scattering angle,  $\Theta \simeq \epsilon/\bar{v}^2$ , is

$$\Delta E(b, \theta, \Phi) = m_2 \gamma_1 \left[ \frac{\epsilon(b)^2}{2\bar{v}(\theta)^4} \left( \frac{\gamma_2}{\gamma_1} - \bar{\gamma} \right) + \frac{\epsilon(b)}{\bar{v}(\theta)^2} v_1 v_2 \gamma_2 \cos \Phi \sin \theta \right]. \quad (18)$$

The differential scattering rate in the cosmic rest frame is also easily worked out from (16),

$$d\Gamma_2(\theta, b, \Phi) = b \bar{v}(\theta) d\bar{n}_1(\theta) db d\Phi. \quad (19)$$

By the assumption of isotropy in the cosmic rest frame

$$d\bar{n}_1(\theta) = \frac{n_1}{2} \sin \theta d\theta, \quad (20)$$

where  $n_1$  is just the number density of species 1.

Finally, the energy transfer rate in the cosmic frame

$$\frac{dE}{dt} = \int \Delta E d\Gamma \quad (21)$$

can be calculated. By integrating over  $\Phi$ , from 0 to  $2\pi$ , the  $\Phi$ -dependent term in (18) averages to zero. The integration over  $\theta$ , from 0 to  $\pi$  can be worked out in the ultra-relativistic limit  $\gamma_1 \gg 1$ . We are left with

$$\frac{dE}{dt} = \pi n_1 \gamma_2 m_2 [\gamma_1^2 - 1 + \mathcal{O}(\gamma_1^{-2})] \int_{b_{UV}}^{b_{IR}} db b \epsilon^2. \quad (22)$$

In the case of interest in this paper we must impose a UV cut-off  $b_{UV} \sim M_D^{-1}$  below which the classical scattering amplitude will not be valid, and at long wavelengths there is a cut-off  $b_{IR} \sim R$  beyond which we recover Newtonian gravity.

## B Energy evolution of the light species

We now want to follow the Energy evolution of the axions down from the temperature  $T_A \sim$  GeV where they are given a mass  $m \simeq 10^{-5}$  eV and they are basically at rest in the reference frame of the cosmological observers. The axions' energy is controlled by two main processes: the gravitational interaction with some relativistic species (electrons, positrons or neutrinos) described in the text and the energy redshift due to the expansion of the Universe:

$$dE = \left( \frac{\mathcal{N}_f M_P}{5\pi d M_D^4} ET^2 - \frac{E}{T} + \frac{m^2}{ET} \right) (-dT) \quad (23)$$

The first term in the parenthesis comes directly from eq. (6), where temperature has been used instead of time ( $t \simeq 10^{-1} M_P/T^2$  during radiation domination) as independent variable and  $n_* = 1.8 T^3/\pi^2$  has been also used. The second and third terms represent the energy loss by redshift. In the range of interest between  $T_A$  and the beginning of matter domination at  $T \simeq 10$  eV the third term is irrelevant. By posing  $E(T_A) = m$  one finds the solution

$$\frac{E(T)}{m} = \frac{T}{T_A} e^{-\mathcal{C}(T^3 - T_A^3)}, \quad (24)$$

where

$$\mathcal{C} \equiv \frac{\mathcal{N}_f M_P}{15\pi d M_D^4} \simeq 10^6 \frac{2\mathcal{N}_f}{3\pi d} \left( \frac{\text{TeV}}{M_D} \right)^4 \text{GeV}^{-3}. \quad (25)$$

The “heating up” process of the axions is very efficient at the beginning and  $E(T)$  reaches a maximum at a temperature which is a fraction of  $T_A$ . From then on, axions cool down by the effect of the redshift; in this regime  $E$  is basically linear in  $T$ :

$$\frac{E(T)}{m} \simeq \frac{T}{T_A} e^{\mathcal{C}T_A^3}, \quad T \ll T_A. \quad (26)$$

Requiring that the axion be non relativistic at  $T_{\text{eq}} = 10$  eV implies requiring the above number to be of order one at that temperature. We obtain  $\mathcal{C}T_A^3 > 20$ , i.e.

$$\frac{\mathcal{N}_f}{\pi d} \left( \frac{T_A}{\text{GeV}} \right)^3 > 3 \times 10^{-5} \left( \frac{M_D}{\text{TeV}} \right)^4. \quad (27)$$

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